



# **A Nondimensional Parameterization for Sound Propagation in the Atmosphere**

**by Michael Mungiole and D. Keith Wilson**

**ARL-TR-2950**

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# **Army Research Laboratory**

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**Computational and Information Sciences Directorate, ARL**

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<b>14. ABSTRACT</b> Parabolic equation (PE) techniques have been successfully used to obtain numerical solutions of sound pressure attenuation in which sound propagation is affected by turbulence and vertical gradients in wind and temperature. The PE models generally produce accurate attenuation values, but the execution time is excessive for applications when near real-time results are required. To obtain sound level attenuation predictions at selected locations more quickly, we are developing an artificial neural network. As a first step in this effort, the PE and boundary conditions were modified to obtain a nondimensional version, written in the MATLAB code. This nondimensional version was developed to be used to train the artificial neural network because a fewer number of parameters (seven) would be required to be specified, resulting in a reduced number of model runs to develop the training algorithm. This report documents the derivation of the appropriate equations that are used in the modified (nondimensional) version of the acoustic propagation model. In addition, graphical data are provided that identify the sensitivity of sound pressure attenuation to each of the seven nondimensional parameters.				
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## 1. Introduction

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Parabolic equation (PE) methods have become very popular for the calculation of sound transmission along ground-to-ground paths in the atmosphere [1]. These methods can account for refraction of the propagating waves by atmospheric wind and sound speed gradients, diffraction into shadow regions, and forward scattering of sound by atmospheric turbulence. With the introduction of the Green's function PE by Gilbert and Di [2], it became possible to perform sound field calculations with parabolic equations very rapidly. Nevertheless, even the Green's function PE is too slow in many Army applications, where calculations are required at many frequencies in fractions of a second. Among these applications are sensor platforms with algorithmic adaption to the propagation environment, tactical decision aids, and combat simulations.

To address the Army's need for faster sound transmission calculations, we are developing an artificial neural network model to predict the transmission loss (diminishment in sound energy resulting from propagation in the atmosphere). If successful, the neural net will provide an extremely fast but still accurate determination of transmission loss. Although neural nets have been used previously in acoustics [3], [4], [5], the main application has been automatic target recognition (ATR). Here, we are attempting something completely different, namely, the prediction of propagation effects. In principle, the propagation neural net could be used to compensate signatures for the propagation effects, thereby improving robustness of ATR algorithms, whether based on neural nets or other methodologies.

In the development of any neural net, an appropriate set of input (training) and output variables must be determined. The output variable for the propagation neural net is simply the sound pressure at the receiver position. The input variables and parameters would be the propagation geometry (target and sensor heights, horizontal separation, sound frequency), atmospheric conditions (wind velocity and sound speed profiles), and acoustical ground properties. Successful development of the neural net requires the output variable to have a fairly smooth dependence on the input variables. It is also desirable to keep the input variables as few in number as possible. For this purpose, we develop in this report a nondimensional version of the acoustic propagation model that describes the propagation with a minimal number of parameters. The propagation model includes the wave propagation equation (in this case, the PE) as well as equations for the boundary conditions, initial condition, and atmospheric fields.

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## 2. Derivation of the Nondimensional Equations

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Among the features of this model are input parameters that specify the acoustic properties of porous media using relaxation theory [6], the ability to handle both narrow and wide angle cases, attenuation of the reflection at the upper boundary, and a second order accurate ground boundary

condition. The model was written in MATLAB<sup>1</sup> [7] and follows the procedures that are outlined in West et al. [1], which use a finite difference numerical technique to solve the parabolic partial differential equation.

## 2.1 Parabolic Equation

We take as our starting point the following narrow angle parabolic equation for sound propagation in a moving, inhomogeneous atmosphere:

$$\partial P / \partial x = i / (2k_0) (\partial^2 / \partial y^2 + \partial^2 / \partial z^2 + k_{\text{eff}}^2 - k_0^2) P, \quad (1)$$

in which  $p = P \exp(ik_0 x)x^{1/2}$  is the sound pressure,  $x$  is the horizontal coordinate in the propagation direction,  $y$  is the horizontal coordinate perpendicular to the propagation,  $z$  is the height,  $k_{\text{eff}}$  is the effective wave number, and  $k_0$  is the wave number at ambient conditions.<sup>2</sup>

Furthermore,  $k_{\text{eff}} = 2\pi f / c_{\text{eff}}$ , in which  $f$  is the frequency and  $c_{\text{eff}} = c + u_x$  is the effective sound speed,  $c$  being the actual sound speed and  $u_x$  the component of the wind velocity in the direction of propagation. This equation includes waves traveling outward from the source in a roughly 30-degree beam. It was derived by Nghiêm-Phu and Tappert [8] and is equivalent to Equation (2.88) in Ostashev [9] if the components of the wind perpendicular to the propagation are neglected.

The PE code we use in this project provides a “pseudo three-dimensional” solution to Equation (1), as described by West et al. [1]. In actuality, the field is determined only in the vertical plane of the  $x$ - and  $z$ - coordinates. This is accomplished by neglecting  $P/x$  in comparison to  $\partial P / \partial x$  (a far-field approximation) and dropping the differentiation in  $y$ , with the result

$$\partial P / \partial x = i / (2k_0) (\partial^2 / \partial z^2 + k_{\text{eff}}^2 - k_0^2) P. \quad (2)$$

This is equivalent to Equation (15) in West et al. [1] (although with  $k_{\text{eff}}$  replacing  $k$ ) and represents the standard narrow angle parabolic equation. Note that  $P$  in this equation does not have units of pressure but rather, units of pressure divided by the square root of length. We will refer to  $P$  as the surrogate sound pressure. In practice, we may set  $k_{\text{eff}} = 2\pi f / c_{\text{eff}} + i\alpha$  in the preceding equation, in which  $\alpha$  is an attenuation coefficient that represents the conversion of sound energy to heat in the propagating waves.

Equation (2) contains three fundamental physical dimensions: time, length, and mass. To develop a nondimensional version of this equation, we must normalize by three quantities that involve these dimensions. We choose here the ambient sound speed ( $c_0$ ), ambient air density ( $\rho_0$ ), and frequency ( $f$ ). Indicating normalized variables with overbars, we have the following nondimensional quantities:

$$\bar{P} = P(c_0 / f)^{1/2} / (\rho_0 c_0^2) \quad (3)$$

$$\bar{x} = xf / c_0 \quad (4)$$

$$\bar{z} = zf / c_0 \quad (5)$$

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<sup>1</sup> MATLAB is a registered trademark of the MathWorks.

<sup>2</sup> *Ambient* in this report refers to an arbitrary reference value that is characteristic of the propagating environment. Typically, the value at the source height is used.

$$\bar{k}_0 = k_o c_0 / f = 2\pi \quad (6)$$

and

$$\bar{k}_{\text{eff}} = k_{\text{eff}} c_0 / f \quad (7)$$

Substituting these quantities into Equation (2) results in the following nondimensional version of the parabolic equation:

$$\partial \bar{P} / \partial \bar{x} = i / (4\pi) (\partial^2 / \partial \bar{z}^2 + \bar{k}_{\text{eff}}^2 - 4\pi^2) \bar{P}. \quad (8)$$

This equation is solved in a manner similar to the procedure used for the dimensional version in West et al. [1], and the detailed steps can be obtained from this reference. First, the terms are rearranged so that the equation is written as

$$\partial \bar{P} / \partial \bar{x} = a \partial^2 \bar{P} / \partial \bar{z}^2 + b_m \bar{P} \quad (9)$$

and the coefficients  $a$  and  $b_m$ , after it is transformed into a finite difference equation, are found to be

$$a = i / (4\pi \Delta \bar{z}^2) \quad (10)$$

$$b_m = i\pi \left[ \left( \frac{\bar{k}_m}{2\pi} \right)^2 - 1 \right] \text{ for } m = 1, 2, \dots, M \quad (11)$$

in which  $M$  is the total number of vertical mesh points and  $\Delta \bar{z}$  is the nondimensional difference between two vertical mesh points

$$\Delta \bar{z} = \bar{z}_{m+1} - \bar{z}_m \quad (12)$$

The complex wave number,  $\bar{k}_m$ , is defined by

$$\bar{k}_m = 2\pi / \bar{c}_{\text{eff}_m} + i\bar{\alpha}_m \quad (13)$$

In this equation,  $\bar{c}_{\text{eff}_m}$  is the effective sound speed in layer  $m$  normalized by  $c_0$  and  $\bar{\alpha}_m$  is the attenuation normalized by wavelength ( $c_0/f$ ).

Once  $\bar{P}$  has been determined, the actual nondimensional pressure follows from the equation

$$\bar{p} = \bar{P} e^{i2\pi \bar{x}} / \sqrt{\bar{x}}. \quad (14)$$

## 2.2 Ground Boundary Condition

As discussed by West et al. [1], the ground impedance boundary condition for the parabolic equation is

$$\partial P / \partial x = -2\pi i f \rho_0 P / Z_c, \quad (15)$$

in which  $Z_c$  is the ground impedance. Many models are available for the impedance of porous ground surfaces such as are encountered outdoors. A particularly simple one that nevertheless gives accurate results was developed by Wilson [10]. The impedance equation is

$$\frac{Z_c}{\rho_0 c_0} = \frac{q}{\Omega} \left[ \left( 1 + \frac{\gamma - 1}{(1 - i\omega\tau_e)^{0.5}} \right) \left( 1 - \frac{1}{(1 - i\omega\tau_v)^{0.5}} \right) \right]^{-0.5} \quad (16)$$

in which

$$\tau_v = \rho_0 q^2 / (2\sigma_p^2 \Omega) \text{ and } \tau_e = N_{pr} \tau_v \quad (17)$$

In these equations, the  $\tau_v$  is a relaxation time associated with viscous diffusion,  $\tau_e$  is a relaxation time associated with thermal diffusion,  $\gamma$  is the ratio of specific heats,  $N_{pr}$  is the Prandtl Number,  $q$  is the tortuosity,  $\Omega$  is the porosity,  $s_p$  is the pore shape factor, and  $\sigma$  is the static flow resistivity. We would like to nondimensionalize the impedance equation in a form using fewer parameters. To do so, we first note that

$$i\omega\tau_v = i2\omega\rho_0 q^2 / (2\sigma_p^2 \Omega) \quad (18)$$

and define a nondimensional flow resistivity,  $\bar{\sigma}$ , which depends on parameters previously defined.

$$\bar{\sigma} = \sigma_p^2 / (f\rho_0 \Omega) \quad (19)$$

Combining Equations (18) and (19) gives

$$i\omega\tau_v = i\pi q^2 / (\bar{\sigma}\Omega)^2 \text{ and } i\omega\tau_e = i\pi N_{pr} q^2 / (\bar{\sigma}\Omega^2) \quad (20)$$

Substituting them into Equation (16) gives

$$\bar{Z}_c = \bar{q} \left[ \left( 1 + \frac{\gamma - 1}{(1 - i\pi N_{pr} \bar{q}^2 / \bar{\sigma})^{0.5}} \right) \left( 1 - \frac{1}{(1 - i\pi \bar{q}^2 / \bar{\sigma})^{0.5}} \right) \right]^{-0.5}, \quad (21)$$

in which  $\bar{Z}_c = Z/(\rho_0 c_0)$  and  $\bar{q} = q/\Omega$ . Considering that  $\gamma$  and  $N_{pr}$  are determined once the temperature is specified, the only parameters that are required to calculate  $\bar{Z}_c$  are  $\bar{\sigma}$  and  $\bar{q}$ .

Thus, the number of ground parameters has been reduced from four ( $\sigma$ ,  $q$ ,  $\Omega$ , and  $s_p$ ) to two ( $\bar{\sigma}$  and  $\bar{q}$ ).

In nondimensional form, the boundary condition, Equation (15), is

$$\partial \bar{P} / \partial \bar{z} = -2\pi i \bar{P} / \bar{Z}_c. \quad (22)$$

Regarding the numerical implementation of  $\partial \bar{P} / \partial \bar{z}$  in spatially discretized form, the first order accurate case can easily be obtained (West et al. [1], Section 5). For the nondimensional case, the first order accuracy boundary condition is found to be

$$\bar{P}_0 = \bar{P}_1(1 - 2\pi i \Delta \bar{z} / \bar{Z}_c)^{-1} \quad (23)$$

and the respective nondimensional boundary condition for second order accuracy is

$$\bar{P}_0 = (4\bar{P}_1 - \bar{P}_2)(3 - 4\pi i \Delta \bar{z} / \bar{Z}_c)^{-1} \quad (24)$$

### 2.3 Upper Absorbing Boundary Condition

In order to avoid spurious numerical reflections of the sound wave at the upper boundary of the computational domain, an absorbing layer at the top of the domain is often added. Salomons [11] suggests the following quadratic form for the attenuation coefficient in this layer:

$$\alpha = \mu \left( \frac{z - z_{\text{abs}}}{z_{\text{top}} - z_{\text{abs}}} \right)^2, \quad z > z_{\text{abs}} \quad (25)$$

in which  $\mu$  is an adjustable parameter (dimensions of inverse length),  $z_{\text{abs}}$  is the height where the absorbing layer begins, and  $z_{\text{top}}$  is the top of the computational domain. The absorbing layer is typically 30 to 100 wavelengths deep. Thus, we might rewrite Equation (25) in nondimensional form as

$$\bar{\alpha} = \bar{\mu} \left( \frac{\bar{z} - \bar{z}_{\text{abs}}}{N_\lambda} \right)^2, \quad \bar{z} > \bar{z}_{\text{abs}} \quad (26)$$

in which  $N_\lambda$  is the depth of the layer in wavelengths,  $\bar{\alpha} = \alpha c_0 / f$ , and  $\bar{\mu} = \mu c_0 / f$ . Salomons [11] suggests values for the parameter  $\mu$  that, while frequency dependent, are only roughly proportional to  $f$ . Here, we propose setting  $\bar{\mu} = 1$ . This leads to values of  $\mu$  that are fairly close to the ones in Salomons' Table I, particularly near 100 Hz.

### 2.4 Initial Condition

The “initial condition” here refers to the sound pressure at  $x = 0$ , from where the solution is marched forward in distance, as opposed to the solution at some initial start time. (Keep in mind that the source is assumed to be harmonic, so that there is no start time.) Following Salomons [11], we use the following Gaussian starter function for the initial condition:

$$P(0, z) = S \sqrt{k_0} \left\{ \exp \left[ -\frac{k_0^2}{2} (z - z_s)^2 \right] + R \exp \left[ -\frac{k_0^2}{2} (z + z_s)^2 \right] \right\}, \quad (27)$$

in which  $z_s$  is the source height,  $S$  is the source strength (in units of pressure) and

$$R = \frac{Z_c / (\rho_0 c_0) - 1}{Z_c / (\rho_0 c_0) + 1} \quad (28)$$

is the surface reflection coefficient. In nondimensional form, these equations become

$$\bar{P}(0, z) = \bar{S} \sqrt{2\pi} \left\{ \exp \left[ -2\pi^2 (\bar{z} - \bar{z}_s)^2 \right] + R \exp \left[ -2\pi^2 (\bar{z} + \bar{z}_s)^2 \right] \right\}, \quad (29)$$

in which

$$\bar{S} = \frac{S}{\rho_0 c_0^2} \quad (30)$$

and

$$R = \frac{\bar{Z}_c - 1}{\bar{Z}_c + 1}. \quad (31)$$

## 2.5 Atmospheric Profiles

By the refraction of sound energy, the wind and temperature profiles near the ground can have a very significant impact on the pressure field. Therefore, we also need a nondimensional model for these profiles. Let us begin by writing the effective sound speed as

$$c_{\text{eff}} = c + u \cos \beta, \quad (32)$$

in which  $c$  is the actual sound speed,  $u$  is the horizontal wind, and  $\beta$  is the angle between the propagation direction and the wind. Neglecting the effect of humidity (which is usually small), the sound speed in air is given by

$$c = c_0 \left( 1 + \frac{T'}{2T_0} \right), \quad (33)$$

in which  $T = T_0 + T'$  is the temperature and  $T'$  a small perturbation about the ambient value  $T_0$ .

The following equations, based on Monin-Obukhov similarity [12, 13], are known to describe the near-ground temperature and wind profiles well over reasonably uniform terrain:

$$T(z) - T(z_r) = \frac{P_t T_*}{k_\nu} \left[ \ln \frac{z}{z_r} - \Psi_h \left( \frac{z}{L_o} \right) + \Psi_h \left( \frac{z_r}{L_o} \right) \right] - \Gamma_d (z - z_r), \quad (34)$$

and

$$u(z) = \frac{u_*}{k_\nu} \left[ \ln \frac{z}{z_0} - \Psi_m \left( \frac{z}{L_o} \right) + \Psi_m \left( \frac{z_0}{L_o} \right) \right], \quad (35)$$

in which  $k_\nu = 0.40$  is von Kármán's constant,  $P_t = 0.95$  is the turbulent Prandtl number,  $\Gamma_d = g/C_p = 0.0098 \text{ K/m}$  is the dry adiabatic lapse rate (accounting for the decrease of temperature with height because of compression in the air column),  $g$  is gravitational acceleration,  $z_0$  is the surface roughness length,  $u_*$  is the friction velocity,  $T_* = -\langle w' T' \rangle_s / u_*$  is a temperature scale, and

$\langle w'T' \rangle_s$  is the covariance of vertical velocity and temperature at the surface (kinematic heat flux). The variable  $z_r$  is a reference height at which the temperature is measured; if we explicitly set  $T_0 = T(z_r)$ , then  $T'$  in Equation (33) equals  $T(z) - T(z_r)$ . The  $\Psi$ 's are universal functions of the dimensionless ratio  $\zeta = z/L_o$ , in which  $L_o = u_*^3 T_0 / k_v g \langle w'T' \rangle_s$  is called the Obukhov length. The following equations for  $\Psi_h$  and  $\Psi_m$  are recommended based on Högström [14] and Wilson [15]:

$$\Psi_{h,m}(\zeta) = \begin{cases} 2 \ln \left[ \left( 1 + \sqrt{1 + a_{h,m} |\zeta|^{2/3}} \right) / 2 \right], & \zeta < 0 \\ -b_{h,m} \zeta, & \zeta \geq 0 \end{cases}, \quad (36)$$

in which the  $a$ 's and  $b$ 's are constants with the values  $a_h = 7.9$ ,  $a_m = 3.6$ ,  $b_h = 8.4$ , and  $b_m = 5.3$ . The case  $\zeta < 0$  corresponds to buoyantly unstable conditions, which typically occur when the sun heats the ground. Buoyantly stable conditions ( $\zeta > 0$ ) typically occur when the ground cools at night. In the limit  $|\zeta| \rightarrow 0$ , the  $\Psi$ -functions equal 0 and the profiles assume their familiar logarithmic forms for neutral conditions.

Now, for the effective sound speed, we have

$$\begin{aligned} \frac{c_{\text{eff}}(z)}{c_0} = 1 - \frac{\Gamma_d(z - z_r)}{2T_0} + \frac{P_t T_*}{2k_v T_0} \left[ \ln \frac{z}{z_r} - \Psi_h \left( \frac{z}{L_o} \right) + \Psi_h \left( \frac{z_r}{L_o} \right) \right] \\ + \frac{u_* \cos \beta}{k_v c_0} \left[ \ln \frac{z}{z_0} - \Psi_m \left( \frac{z}{L_o} \right) + \Psi_m \left( \frac{z_0}{L_o} \right) \right]. \end{aligned} \quad (37)$$

Using the thermodynamic equations  $C_p = C_v + R$  and  $\gamma = C_p/C_v$ , along with the ideal gas law  $c_0^2 = \gamma R T_0$ , we find  $\Gamma_d = g T_0 (\gamma - 1) / c_0^2$ . This relationship allows the second term on the right of the previous equation to be rewritten as  $g(\gamma - 1)(z - z_r) / 2c_0^2$ . Following Wilson [16], it is also convenient to define  $c_* = c_0 T_* / 2T_0$ . We then have

$$\begin{aligned} \frac{c_{\text{eff}}(z)}{c_0} = 1 - \frac{g(\gamma - 1)(z - z_r)}{2c_0^2} \\ + \frac{P_t c_*}{k_v c_0} \left[ \ln \frac{z}{z_r} - \Psi_h \left( \frac{2k_v g c_* z}{u_*^2 c_0} \right) + \Psi_h \left( \frac{2k_v g c_* z_r}{u_*^2 c_0} \right) \right] \\ + \frac{u_* \cos \beta}{k_v c_0} \left[ \ln \frac{z}{z_0} - \Psi_m \left( \frac{2k_v g c_* z}{u_*^2 c_0} \right) + \Psi_m \left( \frac{2k_v g c_* z_0}{u_*^2 c_0} \right) \right]. \end{aligned} \quad (38)$$

Let us next define the following nondimensional variables:  $\bar{c}_{\text{eff}} = c_{\text{eff}}/c_0$ ,  $\bar{c}_* = c_*/c_0$ ,  $\bar{u}_* = u_*/c_0$ ,  $\bar{g} = g/(c_0 f)$ ,  $\bar{z} = zf/c_0$ ,  $\bar{z}_r = z_r g/c_0^2$ , and  $\bar{z}_0 = z_0 g/c_0^2$ :



$$\begin{aligned}
c_{\text{eff}}(\bar{z}) = & 1 - \frac{(\gamma - 1)(\bar{g}\bar{z} - \bar{z}_r)}{2} \\
& + \frac{P_t \bar{c}_*}{k_\nu} \left[ \ln \frac{\bar{g}\bar{z}}{\bar{z}_0} - \Psi_h \left( \frac{2k_\nu \bar{g} \bar{c}_* \bar{z}}{\bar{u}_*^2} \right) + \Psi_h \left( \frac{2K_\nu \bar{c}_* \bar{z}_r}{\bar{u}_*^2} \right) \right] \\
& + \frac{\bar{u}_* \cos \beta}{k_\nu} \left[ \ln \frac{\bar{g}\bar{z}}{\bar{z}_0} - \Psi_m \left( \frac{2k_\nu \bar{g} \bar{c}_* \bar{z}}{\bar{u}_*^2} \right) + \Psi_m \left( \frac{2K_\nu \bar{c}_* \bar{z}_0}{\bar{u}_*^2} \right) \right].
\end{aligned} \tag{39}$$

The profile  $\bar{c}_{\text{eff}}(\bar{z})$  therefore depends on the nondimensional parameters  $\bar{z}_r$ ,  $\bar{z}_0$ ,  $\bar{g}$ ,  $\bar{c}_*$ ,  $\bar{u}_*$ ,  $\cos \beta$ . It may initially seem peculiar to normalize  $z_r$  and  $z_0$  with  $c_0^2/g$  instead of  $c_0/f$ , as was used for other lengths. The advantage of the  $c_0^2/g$  normalization is that  $z_r$  and  $z_0$  are fixed values in many applications; therefore,  $\bar{z}_r$  and  $\bar{z}_0$  are constants and need not be used as parameters in the neural net.

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### 3. Results

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The development of these nondimensional equations results in 13 nondimensional parameters that need to be specified. These are

1.  $\bar{z}_s$ , the source height
2.  $\bar{z}_c$ , the receiver height
3.  $\bar{x}_s$ , the horizontal separation between the source and receiver
4.  $\bar{\sigma}$ , the normalized static flow resistivity for the ground
5.  $\bar{q}$ , the tortuosity to porosity ratio for the ground
6.  $N_\lambda$ , the number of wavelengths in the upper absorbing layer
7.  $\bar{\mu}$ , the attenuation parameter for the upper absorbing layer
8.  $\bar{z}_r$ , the reference height at which the temperature is measured
9.  $\bar{z}_0$ , the roughness length
10.  $\bar{g}$ , the normalized gravitational acceleration
11.  $\bar{c}_*$ , the scale for turbulent sound speed fluctuations
12.  $\bar{u}_*$ , the scale for turbulent wind speed fluctuations (the friction velocity)
13.  $\cos \beta$ , the cosine of the angle between the propagation and wind directions

Many of these parameters would not be needed for the training of a neural net. If the calculations are done properly, the parameters for the upper absorbing layer,  $N_\lambda$  and  $\bar{\mu}$ , should not affect the solution. The reference height  $\bar{z}_r$  is typically a constant value. For a given ground surface,  $\bar{z}_0$ ,  $\bar{q}$ , and the ratio  $\bar{\sigma}/\bar{g}$  are constants.  $\bar{\sigma}/\bar{g}$  is a constant because the source frequency is eliminated when this ratio is taken, resulting in the necessity of only one of these parameters being specified. Thus, as few as seven parameters could be used to train the neural net. These include the three parameters that specify the source and receiver locations ( $\bar{z}_s$ ,  $\bar{z}_c$ , and  $\bar{x}_s$ ), the three parameters needed to calculate the normalized effective sound speed ( $\bar{c}_*$ ,  $\bar{u}_*$ , and  $\cos\beta$ ), and the nondimensional flow resistivity ( $\bar{\sigma}$ ), which is used to calculate the nondimensional ground impedance.

In order to get an idea of the relative importance of each of these seven parameters in generating a training algorithm for an artificial neural network, it would be necessary to determine their sensitivity on the sound pressure attenuation at a specific location. This was performed for the seven parameters, with the results indicated in Figures 1 and 2. The parameter representing each respective curve in these figures was varied over a typical physical range, and the resulting sound pressure attenuation was determined from the nondimensional model. The smallest and largest values for each parameter represent 0% and 100%, respectively, in the figures. These values and the baseline values for the model runs in which the respective parameter was not varied are given in Table 1. For simulation runs in which the receiver height and horizontal separation were not the parameters being varied, the sound pressure attenuation was obtained at a location for which the value of these parameters were 1 m and 500 m, respectively.

The effects of varying the geometric parameters are indicated in Figure 1 while the influence of the parameters used to calculate effective sound speed and ground impedance are shown in Figure 2. Although the range of the sound pressure attenuation is different in these two figures, their scales are equivalent. This allows a direct comparison of the sensitivity of the sound pressure attenuation to each of the seven manipulated parameters. As indicated in these figures, the attenuation is relatively insensitive to the source and receiver heights and friction velocity. For the source and receiver heights, this is attributable to the small range over which these parameters were manipulated. In general, varying static flow resistivity had little effect on sound attenuation. For small values of this parameter, however, there was a more substantial change in the sound pressure attenuation. This could be partly attributable to the large range over which this parameter was varied, which was almost two orders of magnitude. As the wind direction was varied from 0 to 180 degrees (i.e., from the same to the opposite direction of sound propagation), the sound attenuation varied by approximately 10 dB. The ranges over which the normalized horizontal separation and sound speed fluctuation scale were varied had the largest effects on attenuation, with both of them producing more than a 17-dB reduction in the sound pressure. The effect of the sound speed fluctuation scale on attenuation was unique in that it was the only parameter that did not exhibit a monotonically increasing or decreasing relationship.

Because the sound pressure attenuation values are normalized to  $\rho_0 c_0^2$ , it is probably necessary to put the results in proper perspective. Dividing the sound pressure by the product of  $\rho_0 c_0^2$  results in a decrement of approximately 103 dB; thus, one would need to add 103 dB to the values indicated in Figures 1 and 2 to obtain the actual sound pressure level attenuations in the figures.

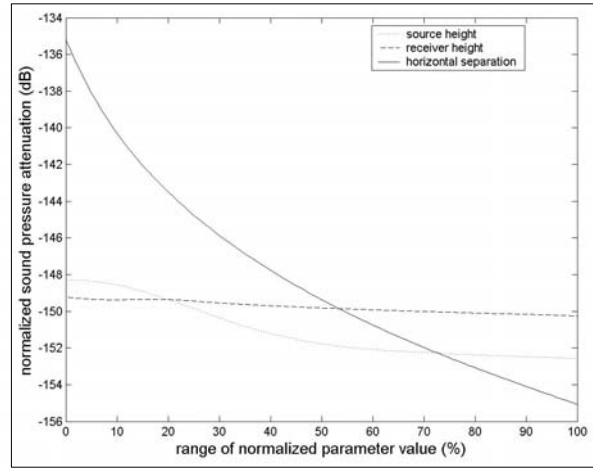


Figure 1. Influence of normalized source height, receiver height, and horizontal separation between them on the normalized sound pressure attenuation.

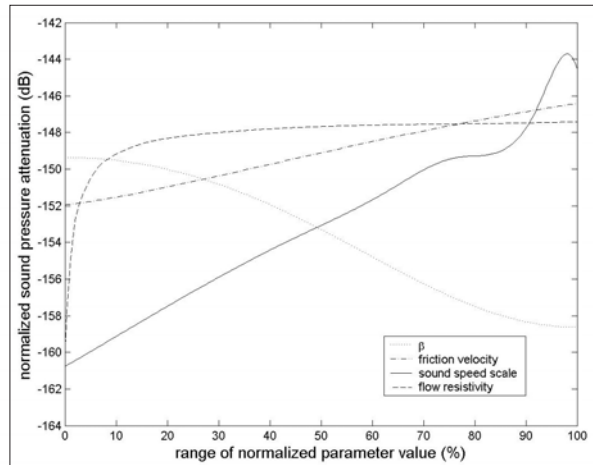


Figure 2. Influence of the angle between propagation and wind directions ( $\beta$ ), and normalized friction velocity, scale for turbulent sound speed fluctuations, and static flow resistivity on the normalized sound pressure attenuation.

Table 1. Range and baseline values for the seven normalized parameters (Actual [dimensional] values and their units are indicated in parentheses.)

Parameter	smallest value	largest value	baseline value
source height, $\bar{z}_s$	0.0	0.74	0.148
(m)	(0.0)	(5.0)	(1.0)
receiver height, $\bar{z}_c$	0.0	0.74	0.148
(m)	(0.0)	(5.0)	(1.0)
horizontal separation, $\bar{x}_s$	14.8	133.4	74.1
(m)	(100)	(900)	(500)
angle between propagation and wind direction, $\beta$	0	$\pi$	0
friction velocity, $\bar{u}_*$	$1.48 * 10^{-4}$	$1.48 * 10^{-3}$	$8.32 * 10^{-4}$
(m/s)	(0.05)	(0.5)	(0.28)
turbulent sound-speed scale, $\bar{c}_*$	$3.5 * 10^{-3}$	$0.296 * 10^{-3}$	$-0.593 * 10^{-3}$
(m/s)	(-1.18)	(0.1)	(-0.2)
static flow resistivity, $\bar{\sigma}$	200	16000	1605
(mks rayls/m)	(25000)	(2000000)	(200000)

## 4. Summary

A system of nondimensional equations for sound propagation in the near-ground atmosphere was developed. They are based on the narrow angle parabolic equation for sound propagation in a moving medium, a relaxation model for the sound-absorbing lower boundary condition, and Monin-Obukhov similarity for the atmospheric profiles.

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